

Structural neurofield mapping: Latent rank model for multivariate data

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1. Introduction

In latent variable models such as factor analysis and the structural equation model (SEM), the latent variables are usually continuous. In SEM, in particular, a latent continuous variable is generally measured by three to five observed variables. However, items in a psychological questionnaire do not have high enough resolution to measure the latent variable on a continuous scale.

Shojima (2008a, 2008b) developed the neural test theory (NTT), which is a latent rank theory. Since tests generally do not have high enough resolution or reliability to evaluate human abilities on a continuous scale, the NTT locates the abilities of examinees on a latent ordinal scale.

In this study, we propose structural neurofield mapping (SNM), which is a latent rank theory for analyzing multivariate data. The SNM model can be said to be something like a confirmatory multidimensional self-organizing map, path analysis models among latent ordinal variables, or an extended multidimensional NTT model.

2. Method

The SNM is a framework that can express various model classes like SEM. For example, the path diagram of a higher-order multidimensional NTT (HOMD-NTT) model is shown in Figure 1. In this figure, the first-order latent rank space is confirmatory three-dimensional, and the second-order latent rank space is one-dimensional. The character located on each latent rank stands for the number of latent ranks of the variable. The NTT model can be said to be the first-order one-dimensional case of the HOMD-NTT model. Therefore, the NTT model is a submodel of the SNM family.

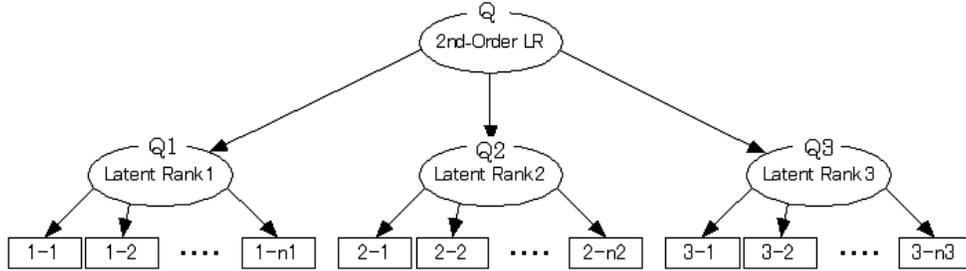


Figure 1: Path Diagram of Higher-Order Multidimensional NTT Model

Let us assume that the number of orders is S and the number of dimensions in the s -th order is M_s . The statistical learning process of the HOMD-NTT model becomes is as follows:

- For $(t=1; t \leq T; t = t + 1)$
- For $(s=1; s \leq S; s = s + 1)$
- For $(m=1; m \leq M_s; m = m + 1)$
- $\mathbf{U}_{sm}^{(t)} \Leftarrow$ Randomly sort the row vectors of \mathbf{U}_{sm} .
- For $(h=1; h \leq N; h = h + 1)$
- Select the winner rank for $\mathbf{u}_{sm,h}^{(t)}$.
- Obtain $\mathbf{V}_{sm}^{(t,h)}$ by updating $\mathbf{V}_{sm}^{(t,h-1)}$.
- $\mathbf{V}_{sm}^{(t+1,0)} \Leftarrow \mathbf{V}_{sm}^{(t,N)}$.

(1)

In the above framework, $\mathbf{U}_{sm}^{(t)}$ is the input data at the t -th period for the m -th latent rank variable in the s -th order, and $u_{sm,h}^{(t)}$ is the h -th row vector of the input data. In addition, $\mathbf{V}_{sm}^{(t,h)}$ is the rank reference matrix after the input vector $u_{sm,h}^{(t)}$ has been learned.

3. Analysis

An English test with a sample size of 3000 was analyzed. This test was composed of six subtests, which contained 6, 16, 7, 5, 5, and 8 items, respectively. We analyzed the data by using a second-order model. The number of dimensions in the first order was six, and the six latent rank variables were set to have 3, 5, 3, 3, 3, and 3 ranks, respectively. In addition, the space of the second order was assumed to be one-dimensional, and the number of ranks of the second-order latent rank variable was set to 10. The category reference profiles of the six lower abilities on the general ability are shown in Figure 2. In addition, the rank membership profiles on the general latent rank scale of the first six examinees out of the 3000 examinees are shown in Figure 3.

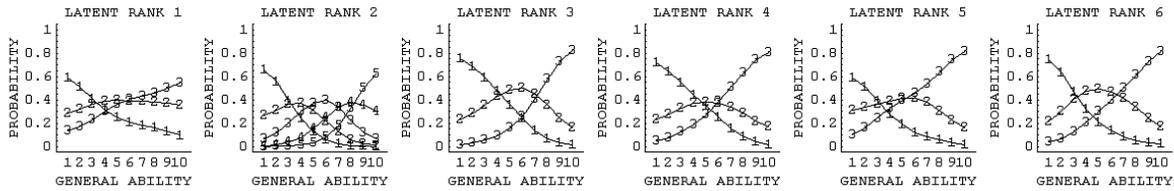


Figure 2: Category Reference Profiles of Six First-Order Latent Rank Variables

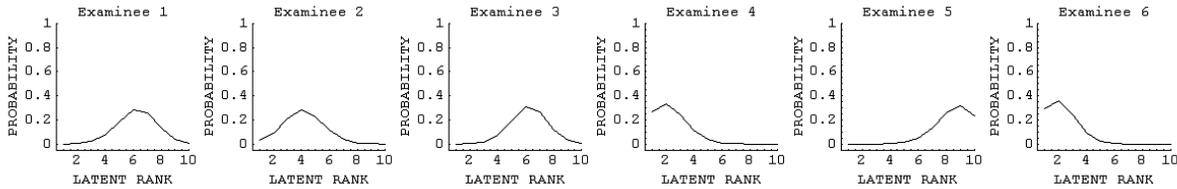


Figure 3: Rank Membership Profiles of First Six Examinees

4. Discussion

In this study, we developed the SNM framework, although we mainly specified the HOMD-NTT model, which is a submodel of the SNM family. The second-order model was effective to simultaneously estimate the lower abilities (the first-order latent ranks) and the general ability (the second-order latent rank).

SNM is like an SEM framework in the case where the latent variables are ordinal. Path analysis modeling or multiple regression modeling among latent rank variables can be expressed in the SNM framework. In addition, multigroup modeling and latent class modeling can be easily added to the framework. Furthermore, a hybrid model in which continuous, ordinal, and nominal latent variables are used simultaneously can be developed by combining the SEM and SNM frameworks.

References

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- Shojima, K. (2008b) Neural test theory: A latent rank theory for analyzing test data. *DNC Research Note*, 08-01.